New Methods for Predictability Analysis

Brian F. Farrell
Harvard University
DEAS
Pierce Hall 107d
Oxford St. Mail Area H0162
Cambridge, MA 02138
phone: (617) 495-2998 fax: (617) 495-9837 email: farrell@deas.harvard.edu

Award Number: N00014-99-1-0018
http://www.fas.harvard.edu/~epsas/people/faculty/farrell

LONG-TERM GOALS

Our long-term goal is to improve operational atmospheric and oceanic forecast capability by maximizing forecast accuracy while minimizing observational and computational costs incurred in producing the forecast.

OBJECTIVES

Our objective is to develop and implement new methods for enhancing forecast capability on two broad fronts. The first is based on advancing optimal statistical estimation theory and developing practical methods for applying it to operational forecast. The challenge is to gain the advantage of optimal state estimation which maximizes the accuracy of the initial state estimate from which the forecast is made while minimizing the a priori overwhelming computational burden optimal estimation methods normally present. Optimal state estimation is particularly attractive because it makes existing observational resources more valuable by extracting the maximum information from them. Our second objective is to develop and implement new methods for optimal ensemble generation for use in ensemble forecast. Ensemble forecast method can improve forecast accuracy and provide error bounds on the forecast but this technique is sensitive to the choice of ensemble members. Our objective is to produce optimal ensembles.

APPROACH

Our approach to developing and implementing optimal state estimation is first to improve fundamental understanding of error dynamics and then use this theoretical base to develop practical methods for computing error statistics so that optimal state estimation algorithms can be efficiently implemented. We accomplish this by reducing the dimension of the error system. Having the objective of practical application of this theory we are particularly interested in developing algorithms for optimal state estimation that use forecast products and algorithms such as the adjoint integrator that is already available at many forecast centers where it is used for operational variational data assimilation.

The theoretical foundation of the method is provided by recent theoretical advances in non-normal time-dependent stability analysis. We use these advances together with methods of modern control

Report Documentation Page

Form Approved
OMB No. 0704-0188

1. REPORT DATE **30 SEP 2001**	2. REPORT TYPE	3. DATES COVERED **00-00-2001 to 00-00-2001**

4. TITLE AND SUBTITLE **New Methods for Predictability Analysis**	5a. CONTRACT NUMBER
	5b. GRANT NUMBER
	5c. PROGRAM ELEMENT NUMBER
6. AUTHOR(S)	5d. PROJECT NUMBER
	5e. TASK NUMBER
	5f. WORK UNIT NUMBER
7. PERFORMING ORGANIZATION NAME(S) AND ADDRESS(ES) **Harvard University,DEAS,,Pierce Hall 107d,Oxford St. Mail Area H0162,Cambridge,,MA, 02138**	8. PERFORMING ORGANIZATION REPORT NUMBER
9. SPONSORING/MONITORING AGENCY NAME(S) AND ADDRESS(ES)	10. SPONSOR/MONITOR'S ACRONYM(S)
	11. SPONSOR/MONITOR'S REPORT NUMBER(S)

12. DISTRIBUTION/AVAILABILITY STATEMENT
Approved for public release; distribution unlimited

13. SUPPLEMENTARY NOTES

14. ABSTRACT
Our long-term goal is to improve operational atmospheric and oceanic forecast capability by maximizing forecast accuracy while minimizing observational and computational costs incurred in producing the forecast.

15. SUBJECT TERMS

16. SECURITY CLASSIFICATION OF:			17. LIMITATION OF ABSTRACT	18. NUMBER OF PAGES	19a. NAME OF RESPONSIBLE PERSON
a. REPORT **unclassified**	b. ABSTRACT **unclassified**	c. THIS PAGE **unclassified**	**Same as Report (SAR)**	**6**	

Standard Form 298 (Rev. 8-98)
Prescribed by ANSI Std Z39-18

theory, specifically balanced Hankel operator truncation methods, to reduce the dimension of the error system. Having reduced the order of the error system we then use the reduced order system to obtain an effective Kalman gain for state identification in the full forecast system. Implementing these methods in operational forecast environments also requires modifying theoretically optimal methods to use available resources to best advantage.

In the second area of our work we are developing theory and methods for incorporating effects of uncertainty in the forecast error system. Uncertainty can arise in the forecast system both from incomplete knowledge of the forecast trajectory and from statistically based parameterization of physics. Ensemble forecast is improved by more accurately accounting for these influences on forecast error growth which are of a different class than the more familiar uncertainties associated with initial conditions. Our approach is to apply recent theoretical advances in non-normal time-dependent stability analysis to the problem of error growth in uncertain forecast systems. Optimal excitation theory will then be used to construct optimal ensembles.

The above described work is a joint effort between Professors Brian Farrell and Petros Ioannou.

WORK COMPLETED

We have completed work on the fundamental theory for error growth in time dependent systems.

We have completed the theory for optimal reduction of the error system order and have shown that error dynamics in these reduced equations accurately models time dependent forecast system error.

We have adapted the control theory approach of optimal balanced Hankel operator model order reduction previously used in association with discrete time independent engineering systems to the time dependent forecast error dynamical system.

We have constructed a reduced order Kalman filter using balanced truncation and demonstrated its ability to accurately estimate the state of the model forecast.

We have developed the theory of error growth in uncertain forecast systems and obtained the error growth and structure under the influence of statistically distributed parameterizations.

RESULTS

We have developed a theory for stability in time dependent systems that facilitates prediction of error statistics and we have shown that the dominant error growth arises intrinsically from destabilization of a restricted set of non-normal vectors of the mean operator by time dependence. Together these results allow us to reduce the dimension of the unstable dynamics of time dependent error systems by representing the dynamics in a similarly restricted subspace.

We have developed a method for reducing the dimension of the time independent error system retaining the dominant error subspace. This method of optimal order reduction of the error system truncates the error dynamics in balanced coordinates in which the stochastic optimals and the empirical orthogonal functions collapse to the same set of structures (Moore, 1981; Glover, 1984; Farrell and Ioannou, 1996).

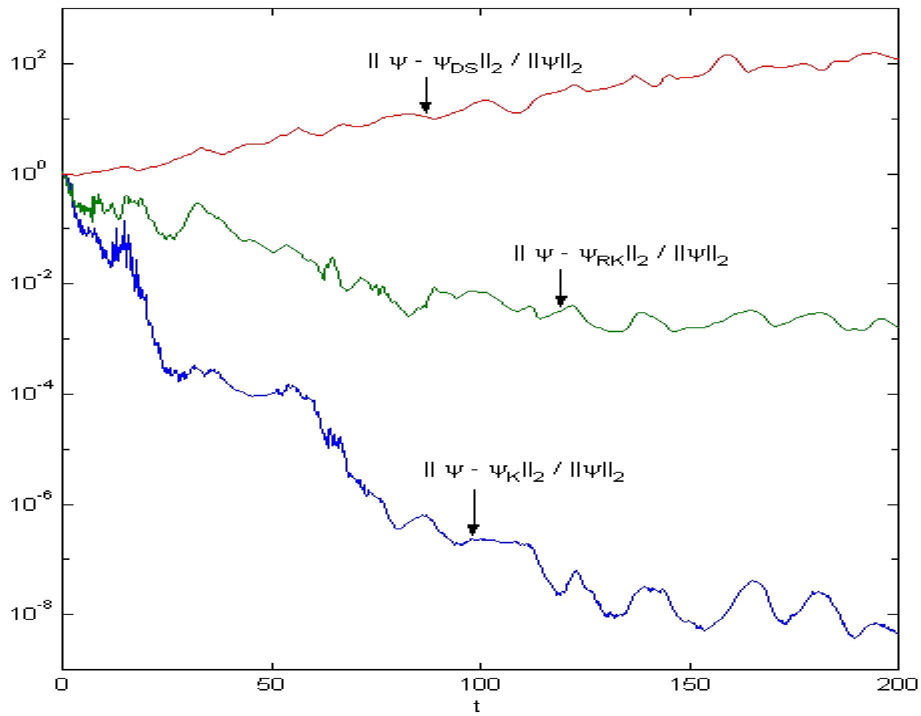

Figure 1: Comparison of the accuracy of 3 methods of state (ψ) identification made using a midlatitude storm track model with 400 degrees-of- freedom (dof) observed at a single height at the jet entry. The full 400 dof Kalman filter is theoretically optimal (bottom curve) but in practice the 3 order of magnitude error reduction achieved by the order 60 approximate Kalman filter is sufficiently accurate for the purpose of practical forecast (middle curve). Present methods of state estimation corresponding to direct substitution fail to identify the state (top curve).

We have implemented and tested the balanced truncation method for reducing the dimension of the forecast error systems in model problems. A very good approximation to the error dynamics is obtained with dimension reduction by as much as a factor of 10 (Farrell and Ioannou, 2001a).

We have determined that truncating the dynamical system in a balanced realization of the optimals and evolved optimals for a single appropriately chosen time provides a nearly optimal reduced order error system. This is a significant result because it suggests that implementing the order reduction algorithm in an operational forecast mode can be greatly simplified.

We have extended the balanced truncation method of optimal order reduction to a time dependent Lyapunov unstable error system and obtained a reduced order Kalman filter in a model time dependent tangent linear forecast system. We showed that this reduced order Kalman filter successfully observes a model of the atmospheric error state (Fig. 1) (Farrell and Ioannou, 2001b).

We have developed the theory of error dynamics in uncertain systems for application to the ensemble forecast problem and obtained exact dynamical equations for the evolution of an ensemble mean field

and the evolution of the ensemble covariance under uncertain dynamics building on previous results obtained for low dimensional systems (Kubo, 1963; Frisch, 1968).

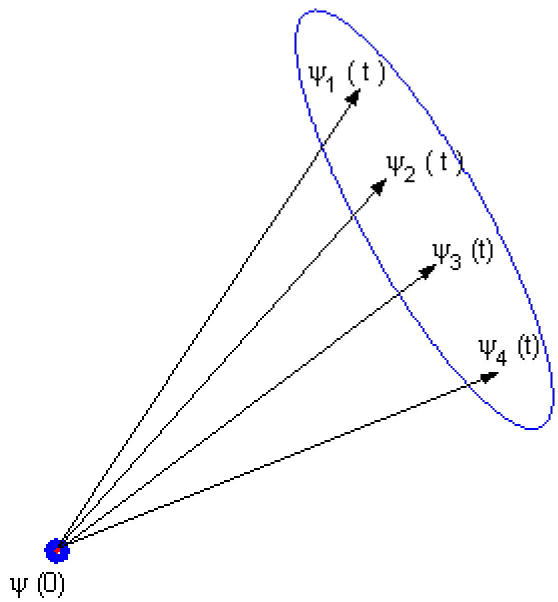

Figure 2: An uncertain forecast error system maps a single initial state to a distribution of states at forecast verification time.

We have solved the problem of optimal excitation for uncertain systems. Because of the uncertainty the future states of the system are entangled (see Fig. 2) and the optimal initial state that produces the greatest expected error growth need not be produced by a single initial condition. We proved that the optimal excitation problem for uncertain systems has a solution: in uncertain systems there is a sure initial condition producing the greatest expected perturbation growth and a sure structure that is most effective in exciting variance when this structure is continuously forced. This result is being extended to provide a method for optimal ensemble generation.

IMPACT/APPLICATION

The methods developed in this work can be directly extended to operational forecast models. Efficient calculation of error statistics using our results allows implementation of advanced methods of state estimation including the Kalman filter.

TRANSITIONS

We have begun implementation of the reduced rank Kalman filter at ECMWF.

RELATED PROJECTS

None

SUMMARY

We have made significant progress toward the goal of increasing forecast accuracy. A fundamental theory for the growth of errors in forecast has been developed and used together with control theory methods to obtain a highly effective approximation to optimal state estimation for the purpose of improving secification of the forecast initial condition. Our method for obtaining more accurate initial conditions has been configured to use operational forecast products and is presently being implemented in an operational forecast at ECMWF.

We are developing a theory for forecast error dynamics that will provide methods for taking account of the effect on forecast error growth and structure of uncertainty in the forecast trajectory and in the forecast model parameterizations.

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